General equations

1. $12x^4 - 4x^3 - 41x^2 - 4x + 12 = 0$ …..(1)

Divide the equation (1) by x^2, then it becomes $12\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)-41=0$ …..(2)

Put $y = x + \frac{1}{x}$, (2) becomes $12(y^2 - 2) - 4y - 41 = 0$

$\Rightarrow\ 12y^2 - 4y - 65 = 0 \quad \Rightarrow\ (6y + 13)(2y - 5) = 0 \quad \Rightarrow\ y = \frac{5}{2}$ or $-\frac{13}{6}$

(a) If $y = \frac{5}{2}$, $x + \frac{1}{x} = \frac{5}{2}$

$2x^2 - 5x + 2 = 0$, $(2x - 1)(x - 2) = 0$

$\therefore\ x = \frac{1}{2},\ 2$

(b) If $y = -\frac{13}{6}$, $x + \frac{1}{x} = -\frac{13}{6}$

$6x^2 + 13x + 6 = 0$, $(2x + 3)(3x + 2) = 0$

$\therefore\ x = -\frac{2}{3}, -\frac{3}{2}$

The roots of (1) are $\frac{1}{2},\ 2,\ -\frac{2}{3},\ -\frac{3}{2}$.

2. Let $f(x) = x^4 + 4x^3 - 18x^2 - 100x - 112$

Put $y = x - k$, or $x = y + k$, then $f(y) = (y + k)^4 + 4(y + k)^3 - 18(y + k)^2 - 100(y + k) - 112$

The y^2 – term $= (6k^2 + 12k - 18)\,y^2$ disappears if $6k^2 + 12k - 18 = 6(k + 1)(k - 1) = 0$

$\therefore\ k = -3$ or 1.

Choose $k = 1$, then $(y + 1)^4 + 4(y + 1)^3 - 18(y + 1)^2 - 100(y + 1) - 112 = 0$

$\Rightarrow\ y^4 + 8y^3 - 120y - 225 = 0, \quad \Rightarrow\ (y^2)^2 - 15^2 + 8y(y^2 - 15) = 0$

$\Rightarrow\ (y^2 - 15)(y^2 + 15 + 8y) = 0, \quad \Rightarrow\ (y^2 - 15)(y + 5)(y + 3) = 0$

$\therefore\ y = -5, -3, \sqrt{15}, -\sqrt{15}$ and $x = y + 1 = -4, -2, 1+\sqrt{15}, 1-\sqrt{15}$.

3. **(a)** $\begin{cases} x^2+y^2+z^2=14 \ldots(1) \\ xy+yz+zx=11 \ldots(2) \\ x+y+2z=9 \ \ldots(3) \end{cases}$

(1) + 2(2), $(x + y + z)^2 = 36 \ \Rightarrow\ x + y + z = \pm 6$ …(4)

(3) – (4), $z = 3, 15$

(i) If $z = 3$, (2) and (3) become $xy + 3(x + y) = 11$ …(5) and $x + y = 3$ ….(6)

(6)↓(5), $xy = 2$ …(7) , Eq. formed from (6),(7) is $t^2 - 3t + 2 = 0$.

Solving we get $(x, y) = (1, 2)$ or $(2, 1)$

(ii) If $z = 15$, (2) and (3) become $xy + 15(x + y) = 11$ …(5’) and $x + y = -21$ ….(6’)

(6’)↓(5’), $xy = 326$ …(7’) , Eq. formed from (6’),(7’) is $t^2 + 21t + 326 = 0$.

Solving we get $(x, y) = \left(\frac{-21+\sqrt{863}i}{2}, \frac{-21-\sqrt{863}i}{2}\right), \left(\frac{-21-\sqrt{863}i}{2}, \frac{-21+\sqrt{863}i}{2}\right)$.

$\therefore\ (x, y, z) = (1, 2, 3),\ (2, 1, 3),\ \left(\frac{-21+\sqrt{863}i}{2}, \frac{-21-\sqrt{863}i}{2}, 15\right), \left(\frac{-21-\sqrt{863}i}{2}, \frac{-21+\sqrt{863}i}{2}, 15\right)$

(b) $\begin{cases} x+y+z=1 & \text{...(1)} \\ x^2+y^2+z^2+6xy=0 & \text{...(2)} \\ \dfrac{x}{y+z}+\dfrac{y}{z+x}+\dfrac{z}{x+y}=0 & \text{...(3)} \end{cases}$

From (1), $y + z = 1 - x$, $z + z = 1 - y$, $x + y = 1 - x$ …(4)

(4) ↓ (3), $\dfrac{x}{1-x}+\dfrac{y}{1-y}+\dfrac{z}{1-z}=0 \Rightarrow x(1-y)(1-z) + y(1-x)(1-z) + z(1-x)(1-y) = 0$

$\Rightarrow x + y + z - 2xy - 2yz - 2zx + 3xyz = 0 \Rightarrow (x + y + z)^2 - 2xy - 2yz - 2zx + 3xyz = 0$, from (1)

$\therefore$ $x^2 + y^2 + z^2 + 3xyz = 0$ …(5)

From (2) and (5), $6xy = 3xyz \Rightarrow xy(z - 2) = 0 \Rightarrow$ $x = 0$ or $y = 0$ or $z = 2$.

(i) When $x = 0$, from (1) and (2) we get $y + z = 1$ …(3) and $y^2 + z^2 = 0$ …(4)

$(4)^2 - (3)$, we have $2yz = 1$ or $yz = 1/2$ …(5)

The eq. formed by (3) and (5) is $2t^2 - 2t + 1 = 0$ $\therefore$ $(y, z) = \left(\dfrac{1+i}{2},\dfrac{1-i}{2}\right)$ or $\left(\dfrac{1-i}{2},\dfrac{1+i}{2}\right)$

(ii) When $y = 0$, using the same method as in (i), we have $(x, z) = \left(\dfrac{1+i}{2},\dfrac{1-i}{2}\right)$ or $\left(\dfrac{1-i}{2},\dfrac{1+i}{2}\right)$.

(iii) When $z = 2$, from (1) and (2) we get $x + y = -1$ ….(6) and $x^2 + y^2 + 6xy = -4$ …(7)

$(7) - (6)^2$, $4xy = -5$ or $xy = -5/4$ …(8)

The eq. formed by (6) and (8) is $4t^2 + 4t - 5 = 0$

$\therefore$ $(x, y) = \left(\dfrac{-1+\sqrt{6}}{2},\dfrac{-1-\sqrt{6}}{2}\right)$ or $\left(\dfrac{-1-\sqrt{6}}{2},\dfrac{-1+\sqrt{6}}{2}\right)$

$\therefore$ (x, y, z) =

$\left(0,\dfrac{1+i}{2},\dfrac{1-i}{2}\right)$, $\left(0,\dfrac{1-i}{2},\dfrac{1+i}{2}\right)$,$\left(\dfrac{1+i}{2},0,\dfrac{1-i}{2}\right)$, $\left(\dfrac{1-i}{2},0,\dfrac{1+i}{2}\right)$,$\left(\dfrac{-1+\sqrt{6}}{2},\dfrac{-1-\sqrt{6}}{2},0\right)$, $\left(\dfrac{-1-\sqrt{6}}{2},\dfrac{-1+\sqrt{6}}{2},0\right)$

(c) Since $x,y,z \neq 0$, put $\begin{cases} u=\dfrac{1}{x} \\ v=\dfrac{1}{y} \\ w=\dfrac{1}{z} \end{cases}$ in $\begin{cases} \dfrac{yz}{y+z}=2 \\ \dfrac{zx}{z+x}=3 \\ \dfrac{xy}{x+y}=1 \end{cases}$ we get $\begin{cases} v+w=\dfrac{1}{2} & \text{...(1)} \\ w+u=\dfrac{1}{3} & \text{...(2)} \\ u+v=1 & \text{...(3)} \end{cases}$

(1) + (2) + (3), we have $2(u + v + w) = 11/6$ or $u + v + w = 11/12$ …(4)

(4) – (1), (4) – (2), (4) – (3), $(u, v, w) = \left(\dfrac{5}{12},\dfrac{7}{12},-\dfrac{1}{12}\right)$

$\therefore$ $(x, y, z) = \left(\dfrac{12}{5},\dfrac{12}{7},-12\right)$

4. **(a)** $x^3 + y^3 = 8$ …(1) ; $x + y = 2$ …(2)

From (1), $x^3 + y^3 = (x + y)^3 - 3xy(x + y) = 2^3 - 3xy(2) = 8 \Rightarrow xy = 0 \Rightarrow x = 0$ or $y = 0$

From (2), $(x, y) = (0, 2)$ or $(2, 0)$

(b) $x + y = 4$ …(1) ; $x^4 + y^4 = 9x^2y^2 + 1$

Since $x^4 + y^4 = (x^2 + y^2)^2 - 2(xy)^2 = [(x+y)^2 - 2xy]^2 - 2(xy)^2 = [4^2 - 2xy]^2 - 2(xy)^2 = 256 - 64xy + 2(xy)^2$

Sub. in (2), $256 - 64xy + 2(xy)^2 = 9(xy)^2 + 1 \Rightarrow 7(xy)^2 + 64xy - 255 = 0 \Rightarrow [7xy + 85][xy - 3] = 0$

$\therefore xy = 3$ or $-85/7$ … (3)

(i) If $xy = 3$, from (1), x, y are roots of $t^2 - 4t + 3 = 0$ …(4)

(ii) If $xy = -85/7$, from (1), x, y are roots of $7t^2 - 28t - 85 = 0$ …(5)

Solve (4) and (5), we get $(x,y) = (1, 3), (3, 1), \left(\frac{14+\sqrt{791}}{7}, \frac{14-\sqrt{791}}{7}\right), \left(\frac{14-\sqrt{791}}{7}, \frac{14+\sqrt{791}}{7}\right)$.

(c) $\frac{x}{5y+1} + \frac{y}{3x+1} = \frac{4}{15}$ …(1) ; $3x + 5y = 2$ …(2)

$\therefore \frac{15x}{5y+1} + \frac{15y}{3x+1} = 4$ …(3) ; $5y = 2 - 3x$ …(4)

(4) ↓ (3), $\frac{15x}{(2-3x)+1} + \frac{3(2-3x)}{3x+1} = 4 \Rightarrow 18x^2 - 9x + 1 = 0 \Rightarrow (3x + 1)(6x - 1) = 0$

$\therefore x = 1/3$ or $1/6$. From (2), $(x, y) = \left(\frac{1}{3}, \frac{1}{5}\right), \left(\frac{1}{6}, \frac{3}{10}\right)$.

(d) $x^3 = 2x + 7y$ …(1) ; $y^3 = 2y + 7x$ …(2)

$\frac{(1)}{(2)}$, $\left(\frac{x}{y}\right)^3 = \frac{2x+7y}{2y+7x} = \frac{2(x/y)+7}{2+7(x/y)} \Rightarrow z^3 = \frac{2z+7}{2+7z}$, where $z = \frac{x}{y}$.

$\therefore 7z^4 + 2z^3 - 2z - 7 = 0 \Rightarrow (z + 1)(z - 1)(7z^2 + 2z + 7) = 0$

$\therefore z = \frac{x}{y} = \pm 1, \frac{-1 \pm 4\sqrt{3}i}{7} \Rightarrow x = \pm y, \frac{-1 \pm 4\sqrt{3}i}{7} y$

(i) When $x = y$, by (1), $x^3 = 2x + 7x$, $x^3 - 9x = 0$, $x(x + 3)(x - 3) = 0$, $\therefore x = -3, 0, 3$.

$(x, y) = (-3, -3), (0, 0), (3, 3)$

(ii) When $x = -y$, by (1), $x^3 = 2x - 7x$, $x^3 + 5x = 0$, $x(x^2 + 5) = 0$, $\therefore x = 0, \pm\sqrt{5}i$

$(\sqrt{5}i, -\sqrt{5}i), (-\sqrt{5}i, \sqrt{5}i)$

(iii) When $x = \frac{-1 \pm 4\sqrt{3}i}{7} y = ky$, by (2),

$y^3 = 2y + 7ky = (2 + 7k)y = \left(2 + 7\frac{-1 \pm 4\sqrt{3}i}{7}\right)y = (1 \pm 4\sqrt{3}i)y = [\pm(2 \pm \sqrt{3}i)]^2 y$

$(x, y) = (0, 0), (2 \pm \sqrt{3}i, 2 \mp \sqrt{3}i), (-2 \pm \sqrt{3}i, -2 \mp \sqrt{3}i)$

$\therefore (x, y) = (0, 0), (-3, -3), (3, 3), (\sqrt{5}i, -\sqrt{5}i), (-\sqrt{5}i, \sqrt{5}i), (2 \pm \sqrt{3}i, 2 \mp \sqrt{3}i), (-2 \pm \sqrt{3}i, -2 \mp \sqrt{3}i)$.

5. $x + \frac{1}{x} = y + 1$ …(1)

$$\frac{(x^2 - x + 1)^2}{x(x-1)^2} = \frac{(x^2 - x + 1)^2}{x(x^2 - 2x + 1)} = \frac{\left[x\left(x + \frac{1}{x} - 1\right)\right]^2}{x^2\left[x + \frac{1}{x} - 2\right]} = \frac{\left(x + \frac{1}{x} - 1\right)^2}{x + \frac{1}{x} - 2} = \frac{y^2}{y-1} \quad \text{….(2)}$$

$(x^2 - x + 1)^2 - 4x(x-1)^2 = 0 \quad \Rightarrow \quad \dfrac{(x^2 - x + 1)^2}{x(x-1)^2} = 4 \quad \Rightarrow \quad \dfrac{y^2}{y-1} = 4 \quad \Rightarrow (y-2)^2 = 0 \quad \Rightarrow \quad y = 2.$

From (1) $\quad x + \dfrac{1}{x} = 2 + 1 = 3 \quad \Rightarrow \quad x^2 - 3x + 1 = 0 \quad \Rightarrow \quad x = \dfrac{3 \pm \sqrt{5}}{2}$ (each double roots)

6. **(a)** $14x^4 - 135x^3 + 278x^2 - 135x + 14 = 0$ ….(1)

Divde the equation by x^2, $\quad 14\left(x^2 + \dfrac{1}{x^2}\right) - 135\left(x + \dfrac{1}{x}\right) + 278 = 0$ ….(2)

Put $y = x + \dfrac{1}{x}$, we get $14(y^2 - 2) - 135y + 278 = 0$, $14y^2 - 135y + 250 = 0$, $(2y-5)(7y-50) = 0$

$\therefore$ $y = 5/2, \quad 50/7.$

(i) When $y = x + \dfrac{1}{x} = \dfrac{5}{2}$, $\quad 2x^2 - 5x + 2 = 0$, $\quad (2x-1)(x+2) = 0$, $\quad \therefore\ x = 2$ or $\dfrac{1}{2}$.

(ii) When $y = x + \dfrac{1}{x} = \dfrac{50}{7}$, $\quad 7x^2 - 50x + 7 = 0$, $\quad (7x-1)(x-7) = 0$, $\quad \therefore\ x = 7$ or $\dfrac{1}{7}$

(b) $8x^4 - 42x^3 + 29x^2 + 42x + 8 = 0$ ….(1)

Divde the equation by x^2, $\quad 8\left(x^2 + \dfrac{1}{x^2}\right) - 42\left(x - \dfrac{1}{x}\right) + 29 = 0$

Put $y = x - \dfrac{1}{x}$, we get $8(y^2 + 2) - 42y + 29 = 0$, $8y^2 - 42y + 45 = 0$, $(4y-15)(2y-3) = 0$

$\therefore$ $y = 15/4, \quad 3/2.$

(i) When $y = x - \dfrac{1}{x} = \dfrac{15}{4}$, $\quad x = 4$ or $-\dfrac{1}{4}$

(ii) When $y = x - \dfrac{1}{x} = \dfrac{3}{2}$, $\quad x = 2$ or $-\dfrac{1}{2}$.

7. $x = \dfrac{a}{b+c},\ y = \dfrac{b}{c+a},\ z = \dfrac{c}{a+b}$ ….(1), $\quad \therefore \quad x + 1 = \dfrac{a+b+c}{b+c},\ y + 1 = \dfrac{a+b+c}{c+a},\ z + 1 = \dfrac{a+b+c}{a+b}$ ….(2)

$\dfrac{(1)}{(2)}$, $\quad \dfrac{x}{x+1} = \dfrac{a}{a+b+c}, \quad \dfrac{y}{y+1} = \dfrac{b}{a+b+c}, \ \dfrac{z}{z+1} = \dfrac{c}{a+b+c}$

Adding up the equalities, $\quad \dfrac{x}{x+1} + \dfrac{y}{y+1} + \dfrac{z}{z+1} = \dfrac{a}{a+b+c} + \dfrac{b}{a+b+c} + \dfrac{c}{a+b+c} = 1.$

8. $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 0 \quad \Rightarrow \quad ayz + bzx + cxy = 0$ ….(*)

Since $\quad (b+c)(-x+y+z)^2 + (c+a)(x-y+z)^2 + (a+b)(x+y-z)^2$

$= (b+c)(x^2+y^2+z^2 - 2xy + 2yz - 2zx) + (c+a)(x^2+y^2+z^2 - 2xy - 2yz + 2zx) + (a+b)(x^2+y^2+z^2 + 2xy - 2yz - 2zx)$

$= 2(a+b+c)(x^2+y^2+z^2) - 4(ayz + bzx + cxy) = \quad 2(a+b+c)(x^2+y^2+z^2) - 4(0) = 2(a+b+c)(x^2+y^2+z^2)$, by (*)

Given expression $= 2(a + b + c)$

9. **(a)** Let $y = \dfrac{x}{x^2 - 5x + 9}$, $\therefore\ yx^2 - (1 + 5y)x + 9y = 0$

For x is real, $\therefore\ \Delta \geq 0$ $\Rightarrow$ $(1 + 5y)^2 - 4(9y) \geq 0$ $\Rightarrow$ $11y^2 - 10y - 1 \leq 0$

$\Rightarrow (11y + 1)(y - 1) \leq 0$ $\Rightarrow$ $-1/11 \leq y \leq 1$

(b) Let $y = \dfrac{x^2 - x + 1}{x^2 + x + 1}$, $\therefore\ (1 - y)x^2 - (1 + y)x + (1 - y) = 0$

For x is real, $\therefore\ \Delta \geq 0$ $\Rightarrow$ $(1 + y)^2 - 4(1 - y)^2 \geq 0$ $\Rightarrow$ $3y^2 - 10y + 3 \leq 0$

$\Rightarrow$ $(3y - 1)(y - 3) \leq 0$ $\Rightarrow$ $-3 \leq y \leq 1/3$

(c) Let $y = \dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$ $\therefore$ $(1 - y)x^2 + (34 - 2y)x + (7y - 71) = 0$

For x is real, $\therefore\ \Delta \geq 0$ $\Rightarrow$ $(34 - 2y)^2 - 4(1 - y)(7y - 71) \geq 0 \Rightarrow$ $y^2 - 14y + 45 \geq 0$

$\Rightarrow$ $(y - 5)(y - 9) \geq 0$ $\Rightarrow$ $y \leq 5$ or $y \geq 9$

10. **(a)** Let $f(x, y) = 3x^2 + 2Pxy + 2y^2 + 2ax - 4y + 1 = 0$

$\Leftrightarrow f(x, y) = 3x^2 + (2Py + 2a)x + (2y^2 - 4y + 1) = 0$

Since $f(x, y)$ can be resolved into linear factors, $f(x, y) = 0$ has rational roots.

$\therefore\ \Delta_x$ must be a complete square.

Consider $\Delta_x = (2Py + 2a)^2 - 4(3)(2y^2 - 4y + 1) = 0$ $\Leftrightarrow (P^2 - 6)y^2 + (2aP + 12)y + (a^2 - 3) = 0$ (*)

Since Δ_x is a complete square, Δ_x has equal roots,

$\therefore\ \Delta_y$ of (*) $= (2aP + 12)^2 - 4(P^2 - 6)(a^2 - 3) = 0$

$\Rightarrow$ P must be one of the roots of the equation $P^2 + 4aP + 2a^2 + 6 = 0$.

(b) $9x^2 + 2xy + y^2 - 92x - 20y + 244 = 0$ $\Leftrightarrow$ $y^2 + (2x - 20)y + (9x^2 - 92x + 244) = 0$

Since y is real, $\therefore$ $\Delta_y = (2x - 20)^2 - 4(1)(9x^2 - 92x + 244) \geq 0$

Simplify, we get $x^2 - 9x + 18 \geq 0$ $\Leftrightarrow$ $(x - 3)(x - 6) \geq 0$ $\Leftrightarrow$ $3 \leq x \leq 6$.

$9x^2 + 2xy + y^2 - 92x - 20y + 244 = 0$ $\Leftrightarrow$ $9x^2 + (2y - 92)x + (y^2 - 20y + 244) = 0$

Since x is real, $\therefore$ $\Delta_x = (2y - 92)^2 - 4(9)(y^2 - 20y + 244) \geq 0$

Simplify, we get $y^2 - 11y + 10 \geq 0$ $\Leftrightarrow$ $(y - 1)(y - 10) \geq 0$ $\Leftrightarrow$ $1 \leq y \leq 10$.

11. **(a)** **(i)** $2x^2 - 9x - 18 < 0$ $\Leftrightarrow$ $(2x + 3)(x - 6) < 0$ $\Leftrightarrow$ $-3/2 < x < 6$

(ii) $12 - 5x - 3x^2 > 0$ $\Leftrightarrow$ $3x^2 + 5x - 12 < 0$ $\Leftrightarrow$ $(x + 3)(3x - 4) < 0$ $\Leftrightarrow$ $-3 < x < 4/3$

(b) $x^2 + 6x + k = (x + 3)^2 + (k - 9)$ is positive for all real x $\Rightarrow k - 9 > 0$ $\Rightarrow$ $k > 9$.

12. $\begin{cases} 1-x_1x_2=0 \\ 1-x_2x_3=0 \\ \quad ...... \\ 1-x_{n-1}x_n=0 \\ 1-x_nx_1=0 \end{cases} \Rightarrow \begin{cases} x_1=1/x_2 \quad ...(1) \\ x_2=1/x_3 \quad ...(2) \\ \quad ..... \\ x_{n-1}=1/x_n \; ...(n-1) \\ x_n=1/x_1 \quad ...(n) \end{cases}$

If n is odd, sub. (2) in (1), $x_1 = x_3$; sub. (3), (2) in (1), $x_1 = 1/x_4$. …. ;

sub (n), (n – 1), …, (2) in (1), we get $x_1 = 1/x_1$. $\therefore\ x_1 = \pm1$

$\therefore\ x_1 = x_2 = \ldots = x_n = \pm1$

If n is even, $x_1 = x_3 = \ldots = x_{n-1} = k$, $x_2 = x_4 = \ldots = x_n = 1/k$ where k is any value.

13. $yz + zx + xy = \frac{1}{2}\left[(x+y+z)^2 - (x^2+y^2+z^2)\right] = \frac{1}{2}\left(c^2 - b^2\right)$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow\ a^3 - 3xyz = c\left[b^2 - \frac{1}{2}\left(c^2 - b^2\right)\right] \Rightarrow\ xyz = \frac{1}{6}\left[2a^3 + c^3 - 3b^2c\right]$

$x + y + z = c$, $yz + zx + xy = \frac{1}{2}\left(c^2 - b^2\right)$, $xyz = \frac{1}{6}\left[2a^3 + c^3 - 3b^2c\right]$

$\Rightarrow$ x, y, z are roots of the equation : $\lambda^3 - c\lambda^2 + \frac{1}{2}\left(c^2 - b^2\right)\lambda - \frac{1}{6}\left[2a^3 + c^3 - 3b^2c\right] = 0$

or $6\lambda^3 - 6c\lambda^2 - 3(b^2 - c^2)\lambda - (2a^2 - 3b^2 c + c^3) = 0$

14. **(a)** $s_1 = x + y + z = m_1$

$s_2 = x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + zx) = m_1^2 - 2m_2$

$s_3 = x^3 + y^3 + z^3 = (x^2 + y^2 + z^2)(x + y + z) - (x + y + z)(yz + zx + xy) + 3xyz = s_2m_1 - s_1m_2 + 3m_3$

$s_4 = x^4 + y^4 + z^4 = (x^3 + y^3 + z^3)(x + y + z) - (x^2 + y^2 + z^2)(yz + zx + xy) + (x + y + z)(xyz)$

$= s_3m_1 - s_2m_2 + s_1m_3$

$s_5 = x^5 + y^5 + z^5 = (x^4 + y^4 + z^4)(x + y + z) - (x^3 + y^3 + z^3)(yz + zx + xy) + (x^2 + y^2 + z^2)(xyz)$

$= s_4m_1 - s_3m_2 + s_2m_3$.

(b) $\begin{cases} x+y+z=3 \\ x^3+y^3+z^3=3 \\ x^5+y^5+z^5=3 \end{cases} \Rightarrow \begin{cases} s_1=3=m_1 \\ s_2=9-2m_2 \\ 3=3s_2-3m_2+3m_3 \\ s_4=9-s_2m_2+3m_3 \\ 3=3s_4-3m_2+s_2m_3 \end{cases} \Rightarrow \begin{cases} m_1=x+y+z=3 \\ m_2=xy+yz+zx=3 \\ m_3=xyz=1 \end{cases}$

$\therefore$ x, y, z are roots of the equation $u^3 - 3u^2 + 3u - 1 = 0$

or $(u - 1)^3 = 0$

or $u = 1$

$\therefore\ x = y = z = 1$